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Heat transfer analysis of intermittent grinding processes

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Abstract

An analytical solution for a two-dimensional boundary-value problem that models the transfer of heat to the workpiece during an intermittent grinding process has been previously constructed. In this solution, two variable functions in the boundary condition of the problem described the interrelation between the grinding wheel, the workpiece and the grinding fluid. In this paper, a numerical algorithm is developed. This algorithm allows one to study the effect on the workpiece temperature of varying either the velocity of the workpiece and/or the cycle-times related parameters. Our objective is to determine the values of the appropriate parameters so that the amount of material removed is maximized and the amount of coolant required is minimized.

Keywords: Grinding process; Boundary-value problem; Analytical solution; Numerical algorithm

1. Introduction

A major problem in grinding is controlling the heat transferred to the workpiece. Part of the energy used in removing stock converts into heat. This effect slows down the grinding process and may cause some thermal damage to the workpiece, such as deformations. The use of grinding fluids helps preventing this thermal damage. They reduce the amount of heat produced at the grinding zone and lubricate the area, thus reducing the friction between the wheel and the workpiece. However, it is well known that these fluids may frequently have a harmful influence on the environment. Therefore, it is of a considerable industrial interest to design devices or improve the process so that the amount of heat transferred to the workpiece is reduced. Currently, it is a common practice to introduce intermittent grinding sequences, either using a slotted wheel or by direct actuation on the workpiece [1,2].

In a previous paper [3], a model and an analytical expression for the evolution of the workpiece temperature field in intermittent grinding was provided. This paper presents an algorithm that allows one to simulate how the workpiece heats during an intermittent grinding process. This model used a boundary-value problem with variable boundary data instead of the classical coupled system of PDEs, see [5–8], which usually receive numerical treatment because of the natural difficulty of finding exact solutions [4,9]. By applying the Green's function method and the convolution theorem for the Laplace transform an explicit solution of the problem in integral form was achieved [3]. The solution was provided in terms of the parameters of the model, thus making it possible to construct an algorithm directly based on it.

The organization of this paper is as follows. In the second section a brief description of the model is provided.

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The same procedure as in Ref. [3] is used to obtain an explicit closed form solution. The third section is a description of the numerical algorithm based on this explicit solution. The composite Simpson's rule is used to approximate the definite integrals and the truncation arguments are physically justified by appealing to the decreasing order of the exponential integrands. The convergence of the algorithm is also shown. The fourth section includes a parametric study of the process, varying the velocity of the workpiece and the duration time of a cycle which includes the contact of the grinding wheel with the workpiece, the cooling of the workpiece surface by interaction with a grinding fluid and the transit of a gap between the wheel and the workpiece along the band of contact. As an example, optimal values of the parameters are given for the particular case of a titanium alloy VT20 workpiece [10] being grounded by an intermittent grinding wheel.

2. Numerical method formulae

In Ref. [3], the physical configuration and the coordinate system depicted in Fig. 1 was assumed for a typical grinding wheel and a workpiece moving in the same direction. The region over which the grinding wheel contacts the workpiece surface is of length δ.

The two-dimensional boundary-value problem

$$\left.\begin{aligned}&\partial_t T(t,x,y) = a(\partial_{xx}T(t,x,y) + \partial_{yy}T(t,x,y)) - v_d\partial_x T(t,x,y),\\ &\lambda\partial_y T(t,x,0) = b(t,x)(T(t,x,0) - T_\infty) + \mathrm{d}(t,x), \quad t \geqslant 0,\\ &T(0,x,y) = T_0, -\infty < x < \infty, \quad y \geqslant 0,\end{aligned}\right\} \tag{1}$$

with variable functions $b(t,x)$ and $\mathrm{d}(t,x)$ in the boundary condition on the surface $y=0$ of the workpiece was used to model the workpiece temperature field during the grinding process mathematically. Considering the change of variables $T(t,x,y) = T(t,x,y) - T_0$ and assuming that $T_0 = T_\infty$, the PDE problem (1) was posed as

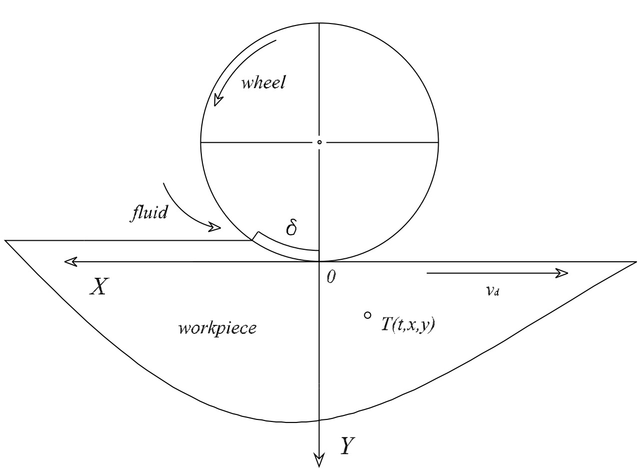

Fig. 1. Grinding setting.

$$\left.\begin{aligned}&\partial_t T(t,x,y) = a(\partial_{xx}T(t,x,y) + \partial_{yy}T(t,x,y)) - v_d\partial_x T(t,x,y),\\ &\lambda\partial_y T(t,x,0) = b(t,x)T(t,x,0) + \mathrm{d}(t,x),\\ &\qquad\qquad -\infty < x < \infty,\ t \geqslant 0,\\ &T(0,x,y) = 0, \quad -\infty < x < \infty,\ y \geqslant 0,\end{aligned}\right\} \tag{2}$$

and the following explicit solution of (2) in integral form was obtained by applying the Green's function method [12] and the convolution theorem for the Laplace transform [13]:

$$\begin{aligned}T(t,x,y) = &\frac{1}{4\pi}\int_0^t \left[\int_{-\infty}^{+\infty} s^{-1} \mathrm{e}^{-\frac{(x'-x-v_d s)^2+y^2}{4as}}\right.\\ &\left.\times\left(\left[\left(\frac{y}{2as} - \frac{b(t-s,x')}{\lambda}\right)T(t-s,x',0) - \frac{\mathrm{d}(t-s,x')}{\lambda}\right]\right)\mathrm{d}x'\right]\mathrm{d}s,\end{aligned} \tag{3}$$

For an intermittent dress grinding process [11], the interaction between the grinding wheel and the workpiece is cyclic. In this case the local heat transfer coefficient $b(t,x)$ and the surface heat flux $\mathrm{d}(t,x)$ were given in [3] in terms of the Heaviside step function H [12]

$$\left.\begin{aligned}&b(t,x) = \alpha H(-x) + \alpha H(x-\delta) + \alpha_s H(\delta - x)H(x)f_\mathrm{s}(t)\\ &\qquad +\alpha H(\delta - x)H(x)f_e(t),\\ &f_\mathrm{s}(t) = \sum_{n=0}^{\infty} H(nt_c + t_p + t_s - t)H(t - nt_c - t_p),\\ &f_e(t) = \sum_{n=0}^{\infty} H((n+1)t_c - t)H(t - nt_c - t_p - t_s),\end{aligned}\right\} \tag{4}$$

and

$$\left.\begin{aligned}&\mathrm{d}(t,x) = -qH(\delta - x)H(x)f_\mathrm{p}(t),\\ &f_\mathrm{p}(t) = \sum_{n=0}^{\infty} H(nt_c + t_p - t)H(t - nt_c).\end{aligned}\right\} \tag{5}$$

In these equations, t_p is the length of the time interval during which contact between the grinding wheel and the workpiece occurs within the nth cycle, t_s is the time during which contact between the workpiece and the grinding fluid takes place in a cycle n and t_c is the duration time of the nth cycle. This includes the contact of the grinding wheel with the workpiece, the cooling of the workpiece surface by interaction with a grinding fluid and the absence of contact along the grinding zone of length δ. In our approximation we suppose that the heat flux due to friction q depends linearly on the workpiece velocity v_d, that is, $q = kv_d$, where k is obtained experimentally [14]. The function f_p describes the contact time between the grinding wheel and the workpiece during the whole process, f_s describes the interaction times between the grinding fluid and the workpiece during the whole process and f_e describes the time period in each cycle when there is no contact. The effective coefficients α and α_s regulate the cooling of the workpiece due to the transfer of heat to the environment and to the grinding fluid respectively.

Finally, taking into account expressions (4) and (5), the explicit solution in (3) was re-written [3] in recurrent form in terms of the Euler error function as

$$T^{(1)}(t,x,y) = T^{(0)}(t,x,y) + \frac{1}{4\pi}\int_0^t \left[\int_{-\infty}^{+\infty} s^{-1} e^{-\frac{(x'-x-v_d s)^2+y^2}{4as}} \times\left(\frac{y}{2as} - \lambda^{-1} b(t-s,x')\right) T^{(0)}(t-s,x',0)\mathrm{d}x'\right]\mathrm{d}s, \tag{6}$$

where

$$T^{(0)}(t,x,y) = \frac{q}{4\lambda}\sqrt{\frac{a}{\pi}}\int_0^t s^{-1/2} f_\mathrm{p}(t-s) e^{-\frac{y^2}{4as}} \times\left(\mathrm{erf}\left(\frac{\delta - x - v_d s}{2\sqrt{as}}\right) + \mathrm{erf}\left(\frac{x+v_d s}{2\sqrt{as}}\right)\right)\mathrm{d}s, \tag{7}$$

is the temperature that the metal workpiece can reach in absence of grinding fluid.

3. Numerical algorithm

Each evaluation of $T^{(1)}$ in Eq. (6) requires the computation of a large number of $T^{(0)}$ terms, as defined in Eq. (7). Therefore, in order to compute $T^{(1)}$ in a reasonable time for engineering purposes, a very quick integration algorithm is needed.

For the sake of simplicity and to avoid unnecessary delays in the computation of the integrals (6) and (7) the composite Simpson's rule is used taking two different subschemas.

First of all, it is essential to find a quick algorithm to compute $T^{(0)}$. If we are interested in the evolution of the workpiece temperature during a time period of length t_end, then the time interval is partitioned into n_1+1 equidistant nodes, $0 = s_0, s_1, \ldots, s_{n_1} = t_\mathrm{end}$. In order to simplify the notation we adopt the values $x = 0$ and $y = 0$ and the equivalence

$$T(t) = T^{(0)}(t,0,0). \tag{8}$$

Taking (7) into account, the integral

$$T(s_1) = \frac{q}{4\lambda}\sqrt{\frac{a}{\pi}}\int_0^{s_1} s^{-1/2} f_\mathrm{p}(s_1 - s) \times\left(\mathrm{erf}\left(\frac{\delta - v_d s}{2\sqrt{as}}\right) + \mathrm{erf}\left(\frac{v_d s}{2\sqrt{as}}\right)\right)\mathrm{d}s, \tag{9}$$

is initially computed. The integrand in the definite integral of (9) has to be tabulated at

$$0 = s_0 = s_0^0, \ldots, s_0^m = s_1, \tag{10}$$

$m+1$ equally spaced points by distance $h = s_1 - s_0$, for m even. In order to evaluate $T(s_2)$ the integrand in integral (9) has to be tabulated at the $2m+1$ equally spaced points

$$s_0 = s_0^0, \ldots, s_0^m = s_1 = s_1^0, s_1^1, \ldots, s_1^m = s_2, \tag{11}$$

and we can use the values computed for the previous integral. Applying this scheme iteratively, the last integral is obtained using $N = m \cdot n_1 + 1$ points, but with a computational cost similar to the one required for the first integral. This is because 95% of the computation time is devoted to the evaluation of the integrands.

The implementation of the method described is straightforward since the integrand in expression (9) is of the form

$$G(t,s) = L(t-s) * M(s). \tag{12}$$

Taking the evaluation of function (12) at any s_j, $j = 0, \ldots, n_1$, equally spaced nodes by distance h, each corresponding integral in (9) is approximated by

$$T(s_j) = \int_0^{s_j} G(t,s)\mathrm{d}s = \frac{h}{3}\sum_{i=1}^{j\cdot(m+1)} C_j L_{N-j(m+1)+i} \times M_i, \tag{13}$$

where $C_j = (1,4,2,\ldots,2,4,1)$ is the coefficient vector for the composite Simpson's rule.

The last step in the algorithm is the approximation of $T^{(1)}$, given by (6). In order to do this, it is necessary to approximate the value of $T^{(0)}$, given by (7), at points $(t-s,x',0)$ at the workpiece surface. Then, we refine the mesh to compute these values by (9) and the improper integral in (6) is truncated to an interval where the exponential factor

$$\phi(s,x') = s^{-1} e^{-\frac{(x'-x-v_d s)^2+y^2}{4as}}, \tag{14}$$

of the integrand is always greater than $1\mathrm{e}-16$.

Moreover, the right endpoint of this interval is always lower than δ, due to the fact that $T^{(0)}(t,x,y) = 0$ if $x > \delta$ for all t and y. Once we have obtained the interval, the value of $T^{(0)}(t-s,x')$ is approximated in the second mesh through a three-point interpolation from the values of $T^{(0)}$ in the first mesh. Knowing $T^{(0)}$, the second integral is also computed using (13).

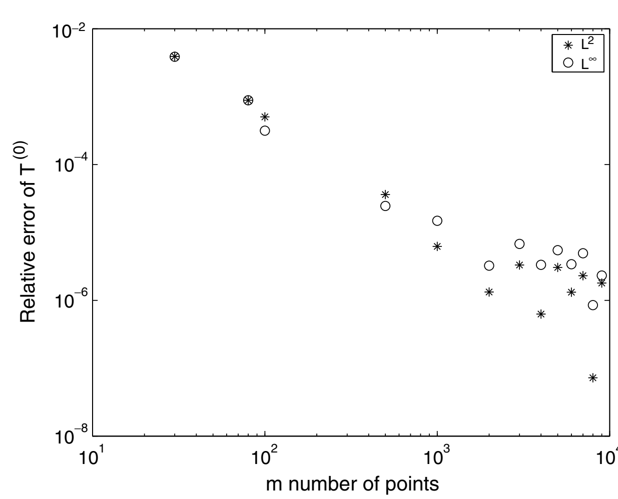

Fig. 2. Relative error of $T^{(0)}$. Note that it is only necessary to use 80 points to reach a 0.1% of relative error compared to the maximum resolution possible in a computer, T_m, with $m = 10000$ points.

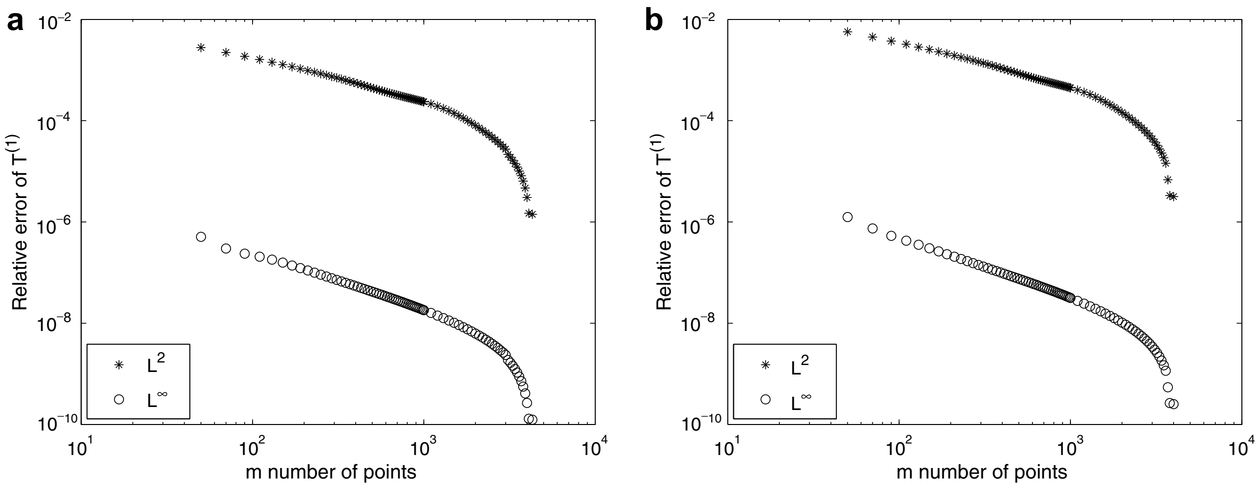

Fig. 3. Relative error of $T^{(1)}$, for $t_s = 0$ (a) and $t_s = 1\mathrm{e}-4$ (b). The relative error is slightly larger in (b).

Finally, we have performed several studies about the convergence of the method. In Fig. 2 we have plotted the convergence results obtained for $T^{(0)}$ in L^2 and L^∞ norms, and the same in Fig. 3 for $T^{(1)}$ in the cases of $t_s = 0$ and $t_s = 1\mathrm{e}-4s$. These results are very satisfactory, since they show that with a very small number of points, we obtain 4–5 digits of precision, more than needed in a usual grinding process.

4. Numerical results

The numerical algorithm has been implemented in MATLAB. The algorithm allows one to simulate the evolution of the workpiece temperature field at every point (x, y) from the rest state until a cyclic regime is reached. The results obtained with the implementation are presented in this section. All the codes used in these simulations are open to the scientific community through a GNU-GPL license. (see Table 1)

The set of parameters that have been used for all further simulations appears in Table 2. These data values have been extracted from Ref. [14] and correspond to an intermittent grinding process of a VT20 titanium alloy metal workpiece.

The effect over the temperature $T^{(0)}$ given by (7) when increasing the workpiece velocity of motion v_d is studied. It is assumed that the heat flux q depends linearly on v_d, see [15]. In Table 3 are given the values of v_d and q used in these simulations.

Fig. 4 shows how the temperature $T^{(0)}$ at the rear edge of the grinding zone (position $(x, y) = (0, 0)$) varies when the velocity is increased. For higher velocities of motion the

Table 1
Nomenclature

Nomenclature			
a	Thermal diffusivity coefficient ($\mathrm{m^2\,s^{-1}}$)	t	Process time (s)
f_e	Process noncontacts function	t_c	nth cycle duration time (s)
f_p	Grinding wheel-workpiece process contacts function	t_s	nth cycle refrigerant application duration time (s)
f_s	Refrigerant-workpiece surface process interaction function	v_d	Workpiece speed ($\mathrm{m\,s^{-1}}$)
n	Cycles index	(x, y)	Position components (m, m)
q	Thermal flux ($\mathrm{W\,m^{-2}}$)	α	Workpiece-environment heat transfer coefficient ($\mathrm{J\,m^{-2}\,K^{-1}\,s^{-1}}$)
T	Workpiece temperature (°C)	α_s	Workpiece-refrigerant heat transfer coefficient ($\mathrm{J\,m^{-2}\,K^{-1}\,s^{-1}}$)
T_0	Inlet temperature (°C)	δ	Grinding zone length (m)
T_∞	Ambient temperature (°C)	λ	Thermal conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$)

Table 2
Input data for numerical simulations

a ($\mathrm{m^2\,s^{-1}}$)	4.23×10^{-6}
q ($\mathrm{W\,m^{-2}}$)	5.89×10^{7}
t_c (s)	1.522×10^{-3}
t_p (s)	1.272×10^{-3}
t_s (s)	0.0
v_d ($\mathrm{m\,s^{-1}}$)	0.53
α ($\mathrm{J\,m^{-2}\,K^{-1}s^{-1}}$)	5.207×10^{4}
α_s ($\mathrm{J\,m^{-2}\,K^{-1}\,s^{-1}}$)	27.29×10^{4}
δ (m)	2.663×10^{-3}
λ ($\mathrm{W\,m^{-1}\,K^{-1}}$)	13

Table 3
Summary of cases

	v_d ($\mathrm{m\,s^{-1}}$)	q ($\mathrm{W\,m^{-2}}$)
■	0.1	1.1113×10^{7}
◆	0.3	3.3340×10^{7}
•	0.53	5.8900×10^{7}
▲	0.7	7.7792×10^{7}

Values of $q = k * v_d$ for different values of v_d. $k = 5.89 \times 10^7/0.53$. The symbols are used in the figures.

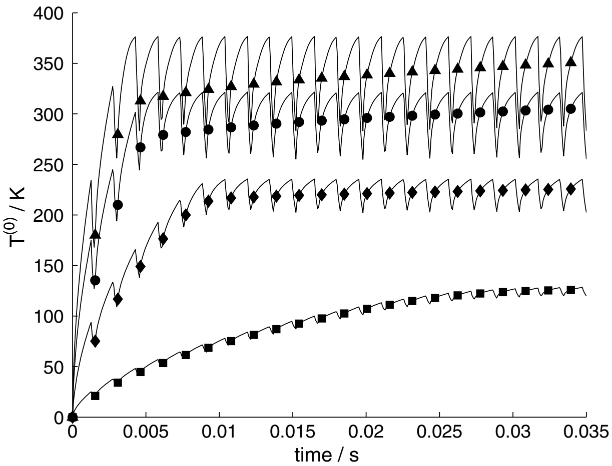

Fig. 4. $T^{(0)}$ expanded in time for several values of v_d and $x = 0 = y$.

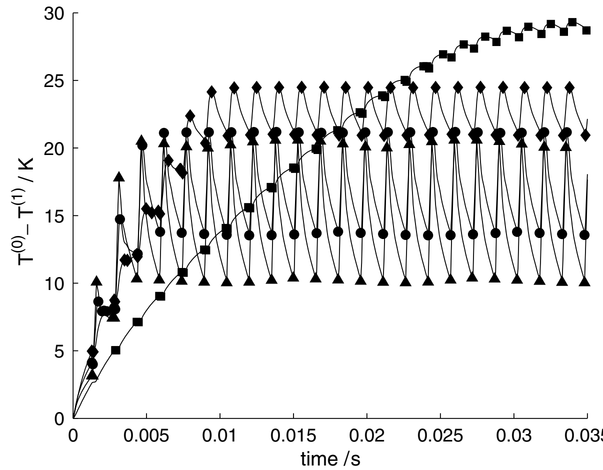

Fig. 7. $T^{(0)} - T^{(1)}$ expanded in time for several values of v_d and $t_s = 1.79\mathrm{e}-4$ at point $(x, y) = (0, 0)$.

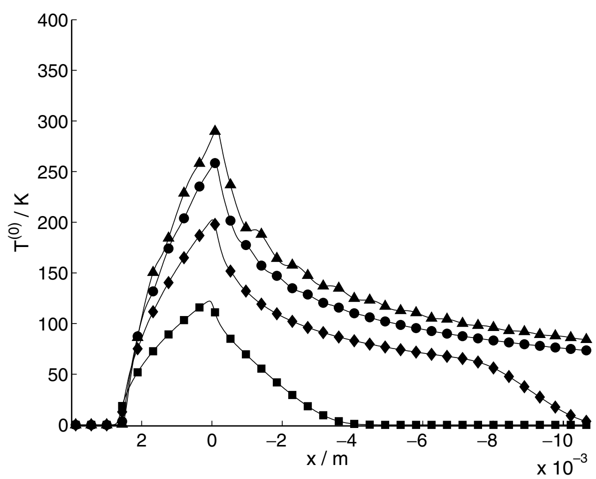

Fig. 5. $T^{(0)}$ expanded in space for several values of v_d.

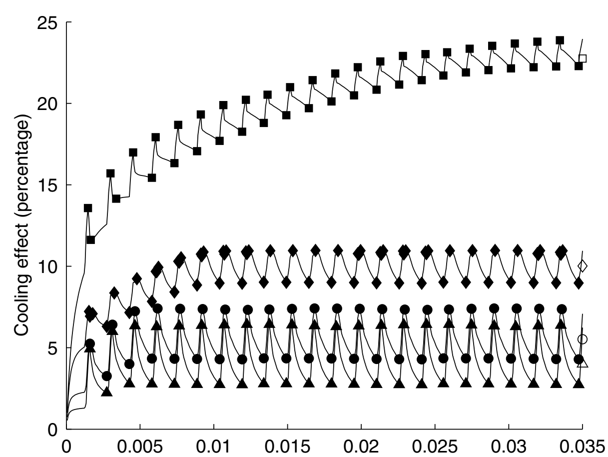

Fig. 8. Cooling effect percentage expanded in time for several values of v_d and $t_s = 1.79\mathrm{e}-4$ at point $(x, y) = (0, 0)$. Open symbols denote the mean values of these curves into the cyclic regime.

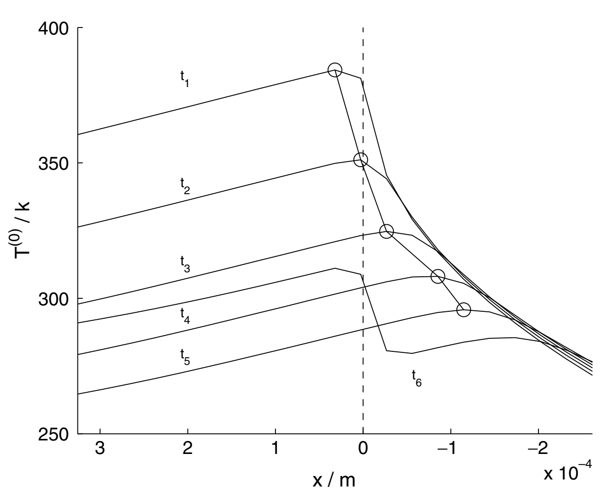

Fig. 6. Zoom of $T^{(0)}$ around $x = 0$ for $v_d = 0.7$. The curves are denoted consecutively in time. t_6 starts a new cycle. The open circles are the local maxima.

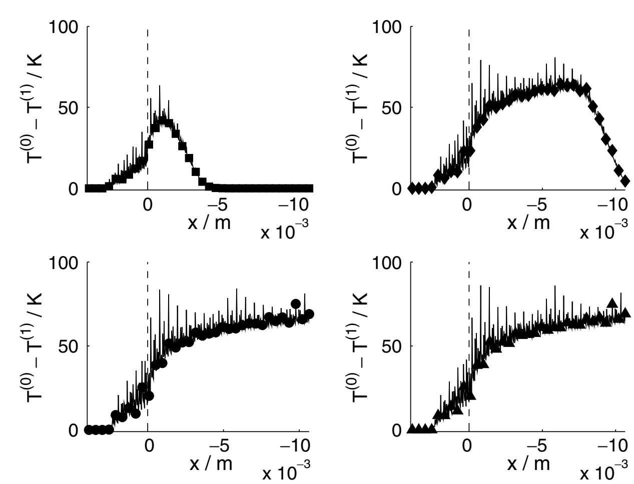

Fig. 9. Evolution of $T^{(0)} - T^{(1)}$ expanded in space for several values of v_d and for $t_s = 1.79\mathrm{e}-4$.

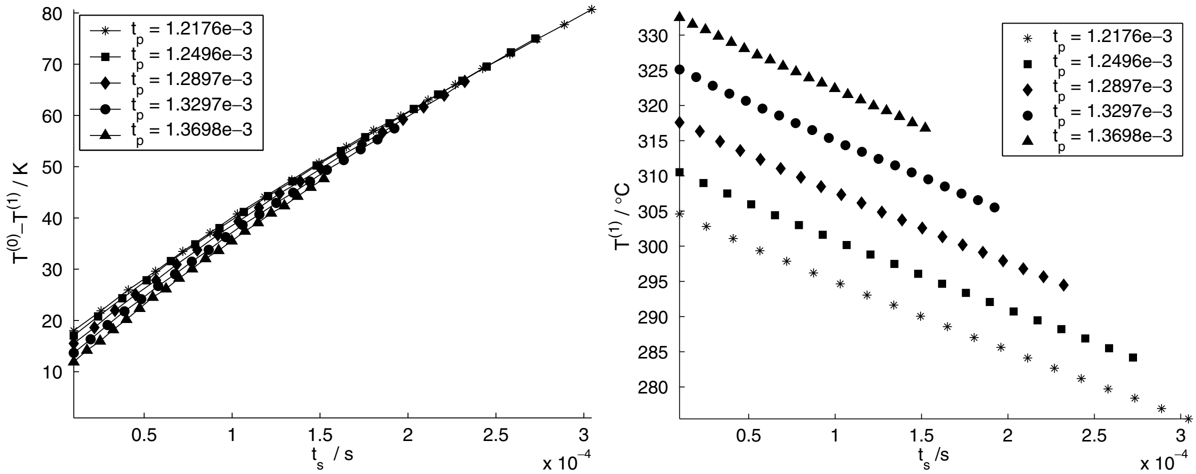

Fig. 10. Evolution of $T^{(0)} - T^{(1)}$ (a) and evolution of $T^{(1)}$ (b) for several values of t_s and t_p.

cyclic regime is reached after a small fraction of time while for values of v_d close to zero the cyclic regime is attained more slowly and the temperature levels decrease considerably.

In Fig. 5 it is pointed out that the maximum values of the workpiece surface $(y = 0)$ temperature are achieved near $x = 0$ for a t in the cyclic regime. The conduction in the direction of the workpiece motion (x) lacks importance and consequently the heating of the workpiece surface in the opposite sense of the workpiece motion is almost imperceptible, with the exception of where the maximum temperature is reached. In the four cases, the maximum of $T^{(0)}$ is not exactly attained at $x = 0$ (Fig. 6), but very close to it, depending on whether there is contact with the wheel or not. When there is contact, this maximum is located still in the grinding zone, being clearly a conductive effect. In the other case the maximum is displaced physically out of this zone. In both cases this maximum is very close to zero.

The evolution of the difference between $T^{(0)}$ given by (7) and $T^{(1)}$ given by (6) according to the velocity v_d of the workpiece motion at point $(x, y) = (0, 0)$ is shown in Fig. 7. It can be observed that the cooling effect saturates after some time. The saturation time decreases with increasing velocity v_d. In Fig. 8 the same effect can be observed, where now the cooling percentage $(T^{(0)} - T^{(1)})/T^{(0)}$ is plotted; mean values are also indicated by empty symbols.

In Fig. 9 the same effect is studied now from a spatial point of view. It is observed that the cooling effect produced by the fluid, and by the non contacts, increases to a maximum located to the right of $x = 0$, that is, once the workpiece is no longer in contact with the wheel but still very close to it.

The effect that the application of grinding fluid has over the temperature reached by the workpiece at point $(x, y) = (0, 0)$ is studied in Fig. 10a and b. Both figures show that from $t_s = 2\text{e} - 4$ s the effect of the length t_p of the time interval during which direct interaction between the wheel and the workpiece surface occurs within the nth cycle is negligible. Therefore, increasing the value of t_p does not make the grinding process more efficient.

Particularly, Fig. 10a shows how the paths of the difference of temperature $T^{(0)} - T^{(1)}$ concentrate forming a curve-sided cone. From approximately $t_s = 2\text{e} - 4$ s the single vertex is reached.

5. Conclusions

In this paper, a numerical algorithm for simulating the evolution of the workpiece temperature during an intermittent grinding process has been designed. The algorithm is based on the explicit solution obtained in a previous paper [3] for a parabolic boundary-value problem in a half-plane. This solution provided a model to approximate the evolution of the temperature field in a metal piece, when grounded intermittently. An implementation of the algorithm in MATLAB can simulate thousands of different configurations in just a few minutes. This makes it possible to determine, for instance, the effect of increasing the amount of refrigerant used. Finally, results for a titanium alloy VT20 real case are presented.

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